
A Constructive Approach of Alexander Duality

Yann-Situ Gazull^{*1}, Alexandra Bac¹, and Aldo Gonzalez-Lorenzo¹

¹Laboratoire d'Informatique et des Systèmes – Aix Marseille Université, Université de Toulon, Centre National de la Recherche Scientifique – France

Résumé

Alexander duality establishes the relation between the homology of an object and the cohomology of its complement in a sphere. For instance, if X is a subset of the 2-dimensional sphere, then each hole of X corresponds to a connected component of its complement in the 2-dimensional sphere, and by symmetry, each hole of the complement of X corresponds to a connected component of X .

In this work, we present a new combinatorial and constructive proof of Alexander duality that provides an explicit isomorphism. The proof shows how to compute this isomorphism using a combinatorial tool called the homological discrete vector field. It also provides a one-to-one map between the holes of the object and the holes of its complement, which we use for representing the holes of an object embedded in \mathbb{R}^3 .

^{*}Intervenant